Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name: Metric Space

Subject Code: 4SC05MSC1 **Branch: B.Sc. (Mathematics)**

Semester: 5 **Date: 27/04/2016** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define: Metric Space. **(02)**

b) Find $int\ A$, $ext\ A$, $fr\ A$,$bd\ A$ for the set $X = N,\ A = \{1, 2, 3, 4, 5, 6\}$. **(02)**

c) Let A and B be any two subsets of a metric space (X, d), then prove that $A \subseteq B$ implies $int\ A \subseteq int\ B$. **(02)**

d) Define: Isometry. **(01)**

e) Define: Complete metric space. **(01)**

f) Define: Closure of a set. **(01)**

g) Is $[5, 6]$ compact? **(01)**

h) Define: Cauchy sequence in a metric space. **(01)**

i) The set Q of rational numbers is closed. Determine, whether the statement is True or False? **(01)**

j) ϕ and X are open in any metric. Determine, whether the statement is True or False? **(01)**

k) Any closed interval with the usual metric is compact. Determine, whether the statement is True or False? **(01)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Prove that the intersection of a finite number of open sets is open. **(05)**

b) Let (X, d) be any metric space. Prove that a subset F of X is closed if and only if its complement in X is open. **(05)**

c) Let l_∞ denote the set of all bounded sequences $\{x_n\}$ of real numbers. Show that the function d define by $d(\{x_n\}, \{y_n\}) = \sup\{|x_n - y_n| : n \in N\}, \forall \{x_n\}, \{y_n\} \in l_\infty$ is metric onl_∞. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Let (X, d) be any metric space. Show that the function d_1 defined by $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)},\ \forall\ x, y \in X$ is a metric on X . **(05)**

	b)	Let A and B be any two subsets of a metric space (X, d). Then prove that $i)\ \overline{A \cup B} = \bar{A} \cup \bar{B}$, $ii)\ \overline{A \cap B} \subseteq \bar{A} \cap \bar{B}$.	**(05)**
	c)	Prove that every closed subset of a compact metric space is compact.	**(04)**
Q-4		**Attempt all questions**	**(14)**
	a)	Which of the following sets are open sets. Justify. $i)\ (-1,1)$ on R, $ii)\ \{ (x,y) / x = y \}$ on R^2, $iii)\ \{(x,y) / x^2 + y^2 < 1 \}$ on R^2, $iv)\ [0,1)$on R, $v)\ (0,1)\ U\ (2,3)$.	**(05)**
	b)	Show that the set $C[a,b]$ of all real-valued functions continuous on the interval $[a,b]$ with the function d defined by $d(f,g) = \left(\int_a^b \left(f(x) - g(x)\right)^2 dx\right)^{\frac{1}{2}}$ is a metric space.	**(05)**
	c)	Prove that every closed sphere is a closed set.	**(04)**
Q-5		**Attempt all questions**	**(14)**
	a)	Let (X, d_1) and (Y, d_2) is metric spaces. Prove that $f: X \to Y$ is continuous if and only if $f(\bar{A}) \subseteq \overline{f(A)}$, for every $A \subseteq X$.	**(05)**
	b)	Let (X, d) be a metric space then show that any disjoint pair of closed sets in X can be separated by disjoint open sets in X.	**(05)**
	c)	Prove that the image of a Cauchy sequence under a uniformly continuous function is again a Cauchy sequence.	**(04)**
Q-6		**Attempt all questions**	**(14)**
	a)	State and prove Banach Fixed point theorem.	**(07)**
	b)	Prove that continuous image of a connected set is connected.	**(05)**
	c)	If $f(x) = x^2$, $0 \le x \le \frac{1}{3}$, then show that f is contracting mapping on $\left[0, \frac{1}{3}\right]$ with the usual metric d.	**(02)**
Q-7		**Attempt all questions**	**(14)**
	a)	Prove that every closed and bounded subset of the real line is compact.	**(07)**
	b)	Let (X, d) be a metric space, and A, B are subset of X, then prove that A is closed if and only if $A \supseteq Fr(A)$.	**(05)**
	c)	Define: Homeomorphism.	**(02)**
Q-8		**Attempt all questions**	**(14)**
	a)	Let (X, d_1) and (Y, d_2) is metric space and f is a function from X into . Then prove that f is continuous at $a \in X$if and only if for every sequence $\{a_n\}$ converging to a, the sequence $\{f(a_n)\}$ should be converges to $f(a)$.	**(07)**
	b)	Prove that continuous image of a compact set is compact.	**(07)**